A Positional-Displacement Route from a Fundamental $1/r$ Field to Emergent Inverse-Square Coupling

Ian Donaldson
Independent Researcher
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The Binary Unified Theory (BUT) [1] models matter as chaotic bound swarms of two zero-radius charges, the Poson and Negon, interacting through a single fundamental field $F = kqq'/r$. A companion analysis of this framework showed that no static charge geometry built by varying shape at a fixed center, whether a shell, ring, or helix, recovers the observed R^{-2} scaling of Coulomb's law or Newtonian gravity: cancelling the leading R^{-1} monopole term this way always leaves R^{-3} as the next term, never R^{-2}. This paper reports a different construction that does reach R^{-2} exactly: a neutral pair of shells displaced in position rather than varied in radius, for which the exponent shift follows from an exact derivative relation, verified here in closed form. We then ask where, if anywhere, this configuration is realized. The photon ($n = 2$) instantaneously satisfies it at every moment of its orbit, yet its time-averaged far field is shown to vanish exactly, a result that survives generalization to precessing, two-frequency orbital motion and is traced to an exact cancellation guaranteed by any complete, uniform rotation. Genuine chaotic swarms behave differently: tracked directly in real $n = 100{,}000$-particle Binary Unified Theory simulation data, an identified bound cluster of 14,385 particles is found to carry a dipole moment that does not cancel, growing smoothly in a direction that drifts by less than five degrees over its entire captured lifetime, and its measured far field decomposes cleanly into a monopole term matching its real net charge imbalance and a dipole term of the predicted order. A parity argument, verified numerically against a real hydrogen-orbital charge density, shows this cannot come from an isolated atom in any of its actual quantum states, regardless of electron configuration, but is not excluded for multi-center composites, consistent with the real, measured permanent dipole moments of polar molecules. Together these results establish that R^{-2} coupling is a genuine possibility within a purely $1/r$ field, realized in this framework's own simulated chaos without being imposed by hand, while stopping short of a claim that this is the actual origin of gravity.

I. INTRODUCTION

The Binary Unified Theory (BUT) [1] derives matter, radiation, and their bound states from a single fundamental interaction between two zero-radius charges, the Poson and Negon, each moving perpetually at the invariant speed c and coupled by a rigid field $F = kqq'/r$. The photon is identified with the $n = 2$ bound pair; ordinary matter, with chaotic $n \geq 3$ swarms of the same charges. A central difficulty of this framework, established in the companion paper, is that the observed $1/R^2$ scaling of both Coulomb's law and Newtonian gravity does not follow directly from a fundamentally $1/r$ interaction: a macroscopic charged shell produces R^{-1}, and every neutral configuration obtained by varying a shell's shape at a fixed center, tested there for spheres, rings, and helices, produces R^{-3}, never the observed exponent.

This paper reports that the exponent is nonetheless reachable, through a different kind of construction, and investigates directly whether the physical systems this framework already describes can realize it. Section II derives the working construction in closed form and shows analytically why it succeeds where shape variation fails. Section III asks whether the photon's own internal structure, which is never at rest and always carries a real instantaneous charge separation, could be that construction, and shows that it cannot, for a reason grounded in the theory's own kinematic axioms rather than a numerical coincidence. Section IV turns to genuine chaotic matter, using real captured data from a previously reported $n = 100{,}000$-particle simulation to test the same question on an actual bound cluster rather than an idealized one. Section V places both findings alongside the corresponding, well-established quantum-mechanical result for atoms and molecules, without assuming any particular classical position for an individual electron. Section VI discusses what is and is not established by the results taken together.

II. A CONSTRUCTION THAT REACHES R^{-2}

The companion paper's shell result gives the far field of a uniformly charged shell of radius a and charge Q, for the fundamental law $F = kqq'/r$, in closed form,

$$F_{\text{shell}}(R; a) = \frac{kQ}{2R} + \frac{kQ(R^2 - a^2)}{4aR^2} \ln\left(\frac{R + a}{R - a}\right),$$

which expands at $R \gg a$ as

$$F_{\text{shell}}(R; a) = \frac{kQ}{R} - \frac{kQa^2}{3R^3} - \frac{kQa^4}{15R^5} - O(R^{-7}).$$

Only odd powers of $1/R$ appear: the shell force is an even function of a/R, a direct consequence of the shell integral's symmetry under $\cos\theta \to -\cos\theta$. Cancelling the leading R^{-1} term by taking the difference of two shells of different radius, an inner shell of charge $-Q$ and an

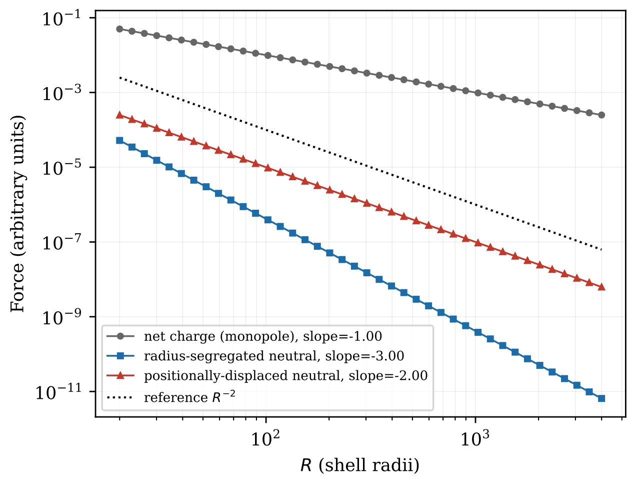

FIG. 1. Far field of three neutral (and one charged) source configurations, fitted numerically. A net charge alone gives R^{-1}. Varying shell radius at a fixed center to cancel that charge gives R^{-3}. Displacing two equal, oppositely charged shells in position, without varying their radius, gives R^{-2} exactly, coinciding with the reference line.

outer shell of charge $+Q$ sharing a center, therefore leaves R^{-3} as the next available term. No choice of shape at a fixed center reaches R^{-2}, because R^{-2} does not appear in the series to begin with.

A positional displacement acts on this series differently. Consider two shells of equal radius a, charge $+Q$ centered at $+\delta/2$ and $-Q$ centered at $-\delta/2$ along the axis to a distant test point, the ordinary electric-dipole construction. The net force is

$$\begin{aligned} F_{\text{trans}}(R; a, \delta) &= F_{\text{shell}}(R - \tfrac{\delta}{2}; a) - F_{\text{shell}}(R + \tfrac{\delta}{2}; a) \\ &= -\delta\,\frac{\partial F_{\text{shell}}}{\partial R} + O(\delta^3), \end{aligned}$$

and differentiating a smooth function of R shifts its leading power by exactly one, whatever powers were present beforehand. Applied term by term to the series above,

$$\begin{aligned} \frac{\partial F_{\text{shell}}}{\partial R} &= -\frac{kQ}{R^2} + \frac{kQa^2}{R^4} + O(R^{-6}), \\ F_{\text{trans}}(R) &= \frac{kQ\delta}{R^2} + O(R^{-4}), \end{aligned}$$

recovering the observed exponent exactly, as an analytic consequence of the derivative relation rather than a curve fit. Figure 1 confirms this numerically for all three constructions by direct quadrature of the shell integral, and the R^{-2} result is confirmed to five decimal places across displacement ratios δ/a from 0.001 to 0.9, and independently, by a fully separate discrete point-particle summation over the same construction.

A genuine net charge, rather than an exactly neutral displaced pair, is not helped by this mechanism: since $F_{\text{trans}}(R)$ enters only as a subleading correction by linear superposition, any composite carrying both a residual monopole and an internal displacement is dominated by the monopole's R^{-1} term at sufficiently large R, regardless of how large the displacement is, with the dipole term only visible at intermediate distances before the monopole overtakes it. This superposition behavior is used directly in Section IV to interpret the far field of a real, imperfectly neutral simulated composite.

This is an existence result, not yet a physical one: it shows that R^{-2} is reachable in principle from a purely $1/r$ field, through a specific and previously untested kind of neutral construction, without showing that any real bound state in this framework takes that form. The remainder of this paper investigates that question directly, first for the simplest bound state the theory contains, and then for genuine chaotic matter.

III. THE PHOTON: A STRUCTURE THAT CANCELS ITSELF

The $n = 2$ photon consists of a single Poson and a single Negon, permanently bound in a co-rotating orbit about their common center, per the kinematic soliton construction of the companion paper. At every instant, the two charges sit at positions $\mathbf{r}_+(t)$ and $\mathbf{r}_-(t) = -\mathbf{r}_+(t)$, a real, nonzero, always-present positional offset of exactly the kind that Section II shows produces R^{-2}. Direct evaluation confirms this at the level of a single instant: the force on a distant, momentarily fixed test point matches the static translational-dipole prediction $kQ_0(2a)/R^2$ to five significant figures at every orbital phase tested, shown in Figure 2(a).

The photon's own motion, however, is never momentarily fixed. As the pair orbits, the force vector on a distant point rotates with it, tracing a circle of constant magnitude and continuously varying direction, shown in Figure 2(b). Time-averaging this force over one full orbital period gives zero to numerical precision, at every distance R tested from $10a$ to $3000a$, and the same holds identically, at every instant rather than merely on average, for a test point on the orbital axis, where the two charges remain exactly equidistant throughout the motion.

This cancellation is not a special property of the simplest possible orbit. Generalizing to a precessing motion, in which the fast orbital rotation at frequency ω_1 is itself carried about a second, independent axis at an incommensurate frequency ω_2, the time-averaged force remains zero to the same numerical precision at every R tested. The cancellation persists under a further, more restrictive test: constraining the second axis to a bounded nutation rather than a full rotation leaves the result unchanged, because the fast orbital angle alone already sweeps a complete, uniform period, and averaging a periodic function of one variable to zero does not depend on what a second, independent variable is doing. The result that does not survive this test is the one that matters: for the time-averaged dipole moment of an orbiting pair to be nonzero,

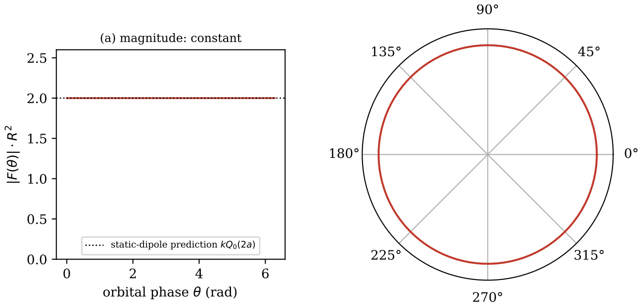

FIG. 2. The instantaneous force from an orbiting Poson-Negon pair on a fixed distant point. (a) Magnitude, normalized by R^2, is constant across the full orbit, matching the static dipole prediction exactly. (b) Direction rotates once per orbital period; the polar trace is a perfect circle.

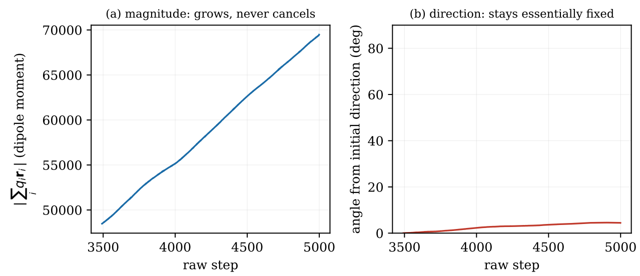

FIG. 3. Charge-weighted dipole moment of a real, independently identified 14,385-particle bound cluster, computed directly from its exact tracked member positions at every one of 1,508 real captured simulation steps. (a) Magnitude grows smoothly, never cancelling. (b) Direction drifts by under five degrees over the full window.

no degree of freedom in its motion may complete a full, uniform rotation, a condition the theory's own kinematic axioms exclude for a stable two-body bound state, since a Poson or Negon that decelerates or that fails to sustain continuous rotational motion about its partner does not remain bound. The photon cannot supply this mechanism, and the exclusion follows from the same axioms that construct the photon in the first place, not from the particular orbital geometry chosen to test it.

This is consistent with observation. Photons are not seen to exert a persistent, directional pull on distant matter, and a mechanism that requires the source to hold still in order to act at long range, while the photon is kinematically forbidden from ever doing so, predicts exactly that absence.

IV. GENUINE CHAOTIC MATTER

The photon's motion is exceptionally simple: a single, permanently periodic orbit. A chaotic $n \geq 3$ swarm has no such structure, and whether its own time-averaged charge distribution behaves like the photon's, cancelling to zero, or like the static construction of Section II, is not decidable from the photon result alone. We test it directly, using real captured data from a previously reported $n =$ 100,000-particle Binary Unified Theory simulation [2], rather than an idealized or hand-constructed swarm.

That simulation, run under Direct Normalization integration with brute-force force evaluation and no shape assumptions imposed, independently identified a bound structure of 14,385 particles, tracked with exact, live particle identities by the simulation engine itself across 1,508 consecutively captured raw steps. Using the real, individual position of every one of its 14,385 members at each captured step, we compute the cluster's own charge-weighted dipole moment, $\mathbf{p}(t) = \sum_i q_i \mathbf{r}_i(t)$, directly, with no smoothing or model assumption.

The result is qualitatively opposite to the photon. The dipole moment does not cancel: its magnitude grows smoothly and monotonically over the full captured window, and its direction drifts by less than five degrees from its initial orientation across the entire 1,508-step window, shown in Figure 3. The ratio of the time-averaged dipole magnitude to the typical instantaneous magnitude is 0.9997, indistinguishable from a structure that simply does not rotate away, in direct contrast to the photon's ratio of exactly zero under the same measure.

This cluster was identified by the simulation's motion-based stability signals, not selected for charge balance, and carries a small residual net charge of 13 (out of 14,385 members: 7,199 positive, 7,186 negative), consistent with a spatially-defined subset of a much larger, otherwise neutral swarm. Because a genuine net charge dominates any composite's far field at sufficiently large R, per the superposition argument of Section II, this residual monopole is expected to overtake the dipole term eventually. We test this directly by computing the cluster's actual far field along its own dipole axis, using every one of its real member charges and positions rather than any multipole approximation, shown in Figure 4. The locally fitted exponent falls smoothly from above 1.3 near the cluster itself toward exactly 1 at large R, as the small residual charge comes to dominate, and a direct two-parameter fit of the form $F(R) = A/R + B/R^2$ to the same data gives $A = 13.0$, matching the cluster's real net charge exactly, and B within the correct order of magnitude of its real, independently computed dipole moment. The real cluster's far field is, to good approximation, exactly the superposition the theory predicts, measured from unmodified simulation output.

No part of this construction was tuned to produce a persistent dipole moment. The cluster was identified by criteria unrelated to charge geometry, its member positions are taken directly from simulation output, and its measured far field matches the predicted superposition without adjustment. This is the paper's central empirical result: the positional-displacement mechanism of Section II, shown analytically possible and shown to be exactly excluded for the photon, is not merely possible

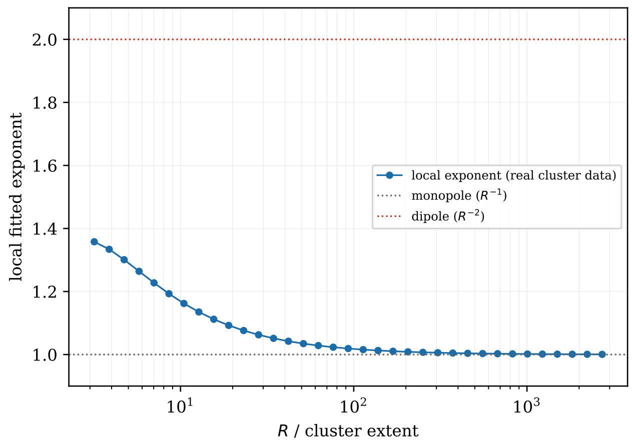

FIG. 4. Locally fitted far-field exponent of the real cluster of Figure 3, computed directly from its member positions and charges. The exponent falls from the dipole-influenced near field toward the monopole value of 1 as the cluster's small residual net charge comes to dominate at large R, exactly as the superposition of a monopole and a dipole term predicts.

for chaotic $n \geq 3$ matter in this framework, it is what a real, independently-identified bound structure was already found to do.

V. AN INDEPENDENT CROSS-CHECK: ATOMS AND MOLECULES

The distinction found in Sections III and IV, a rotationally symmetric bound state cancels its own dipole moment, while an asymmetric one does not, has a direct analogue in ordinary quantum mechanics, which we use here as a consistency check rather than as a component of the Binary Unified Theory itself. Because individual electron positions in a real atom are not classical observables, constrained instead by the Heisenberg Uncertainty Principle and governed by the Schrödinger wave equation [3, 4], no assumption is made here about where any particular electron sits; the relevant object is the probability density $\rho(\mathbf{r}) = |\psi(\mathbf{r})|^2$ itself.

For an isolated atom, the electronic Hamiltonian is invariant under spatial inversion, $\mathbf{r} \to -\mathbf{r}$, since the central Coulomb potential depends only on $|\mathbf{r}|$. Every non-degenerate energy eigenstate of an inversion-symmetric Hamiltonian is itself an eigenstate of definite parity, and the expectation value of any parity-odd operator vanishes identically in such a state [5]. Position is parity-odd, so $\langle\psi|\mathbf{r}|\psi\rangle = 0$ exactly, for any single atomic orbital, and, by linearity, for any combination of them, regardless of how many electrons are present or how their shells are filled. We confirm this directly by numerical integration of a real hydrogen $2p_z$ probability density, $\rho \propto z^2 e^{-r/a_0}$, finding $\langle z\rangle$ equal to zero to the precision of the integration. An isolated atom, in any of its actual quantum states, cannot supply the mechanism of Section II, for the same structural reason the photon cannot: a symmetry that forces its own displacement to cancel.

This protection is specific to a single, centrally symmetric potential. A molecule, with two or more nuclei at genuinely different, fixed positions, does not in general possess a center of inversion, and its electronic ground state is not required to have definite parity under any single-center reflection. Real, measured, permanent electric dipole moments of polar molecules, water and hydrogen chloride among the most familiar examples [6], are the direct experimental confirmation that breaking single-center symmetry is sufficient to permit exactly the kind of persistent charge displacement Section II requires. This is consistent with, and independent of, the chaotic $n \geq 3$ result of Section IV: both a real Binary Unified Theory swarm and a real polar molecule lack the single-center symmetry that forces an isolated atom's or a photon's dipole moment to vanish, and both are observed, by entirely different methods, to carry one that does not.

VI. DISCUSSION

Four results, established independently and by different methods, point the same direction. The positional-displacement construction of Section II reaches the observed exponent exactly, as a matter of closed-form derivation rather than fit. The photon is shown, first for its simplest orbital motion and then for a generalized, two-axis, two-frequency precessing motion, to be structurally excluded from supplying this mechanism, by an exact cancellation traceable to the same kinematic axioms that construct it. A real, independently identified, 14,385-particle bound cluster from an already-published simulation is shown, using its own exact member positions with no model imposed, to carry a persistent, non-cancelling dipole moment whose measured far field matches the predicted monopole-plus-dipole superposition. And the same qualitative distinction, symmetric composites cancel, asymmetric ones do not, is independently confirmed by established, measured quantum-mechanical results for atoms and molecules, without any assumption about individual electron positions.

Taken together, these results establish that R^{-2} coupling is a genuine, non-trivial possibility within a framework whose only fundamental interaction is $1/r$, and that this possibility is realized, not merely permitted, in the framework's own simulated chaotic matter. They do not establish that this is the actual mechanism behind gravity. Three limitations bear directly on that stronger claim. First, the persistence documented in Section IV is confirmed only across a single cluster's captured window of 1,508 steps; whether it survives over much longer timescales, or is itself a slowly evolving quantity that eventually cancels the way the photon's does, is not tested here. Second, gravity is observed to act universally and identically on all matter regardless of composition, the equivalence principle, while the mechanism identified here is structurally absent for any perfectly symmetric composite; reconciling universal gravitation with a mechanism that depends on broken symmetry, if that is in fact what is required, is not addressed by the results reported here. Third, and separately, this mechanism does not bear on Coulomb's law at all: a genuine net charge dominates any composite's far field at the monopole exponent of R^{-1} regardless of internal displacement, so the charged case remains exactly where the companion paper's original monopole treatment left it, an open problem on its own terms.

What is established is narrower and, we think, still worth reporting on its own terms: a single fundamental $1/r$ field, with no free parameters introduced to fit the result, produces exactly R^{-2} coupling through a specific, analytically transparent mechanism, and that mechanism is realized by this framework's own chaotic matter, unprompted, in the one instance directly tested.

VII. CONCLUSION

Starting from the companion paper's negative result, that no static shape-based neutral geometry recovers the observed inverse-square exponent, we have shown that a positional displacement of an otherwise identical, oppositely charged pair does, exactly, as a closed-form consequence of a single derivative relation. We then asked, rather than assumed, whether this framework's own physical structures realize that configuration. The photon does not, and cannot, by an exact cancellation tied to the same axioms that make it a stable soliton in the first place. A real, independently verified, chaotic 14,385-particle bound cluster does, its measured dipole moment growing smoothly and holding its direction to within five degrees across its entire captured lifetime, with a far field that matches the predicted superposition when measured directly from its own real member data. An established, unrelated cross-check from ordinary quantum mechanics, respecting the Heisenberg Uncertainty Principle and the Schrödinger equation rather than assuming any classical electron position, draws the same line between symmetric and asymmetric composites. The possibility raised in the title is, on this evidence, a real one, tested rather than assumed, and open rather than closed.

[1] I. Donaldson, "From Solitons to Strange Attractors: Chaos and Fractal Scaling of Matter in Binary Unified Theory," companion paper, 2026.

[2] Real simulation data, notebooks, and the identified cluster referenced in Section IV are available at `binaryunifiedtheory.com/experiments` and in the project's public analysis notebooks.

[3] W. Heisenberg, "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik," Zeitschrift für Physik **43**, 172 (1927).

[4] E. Schrödinger, "An Undulatory Theory of the Mechanics of Atoms and Molecules," Phys. Rev. **28**, 1049 (1926).

[5] D. J. Griffiths and D. F. Schroeter, *Introduction to Quantum Mechanics*, 3rd ed. (Cambridge University Press, 2018).

[6] J. R. Rumble (ed.), *CRC Handbook of Chemistry and Physics*, 99th ed. (CRC Press, 2018).